Transconductance Realization of Block-diagrams of Electronic Networks

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Abstract—A method of realization of block-diagram is proposed. The algorithm of transconductance realization is presented. The features of the algorithm for the design of high-precision amplifiers is discussed. The results of circuit simulation of electric networks based on the transconductance amplifiers, and a comparison between these networks and conventional operational amplifiers based circuits are included.

Keywords: block-diagram, transconductance amplifier (transconductor), operational amplifier, signal-flow graphs, matrix, extraction of a branch or a controlled source, MOS design, circuit simulation.

I. INTRODUCTION

The block-diagrams represents physical connections between the real elements of electronic networks or systems, which define information, functional and energy transformations of the signals. They are often used on all three main stages of the network design: systematic, circuit and structural. For example on Fig. 1 presented the block-diagram of amplifier with additive correction [1]. The transfer function of amplifier is described as

\[ S = K_1 + K_2 \beta - K_1 K_2 \mu. \]  

(1)

Fig. 1. The block-diagram of the amplifier with additive correction

The process of the network design, as any creative work, is poorly formalized, that is why so much important that transition from one stage of designing of electronics to other must be transparent and unambiguous. Unfortunately, this requirement is difficult to accomplish at realization of the block-diagrams by electronic network with usage of traditional methods and conventional elements.

So, in the Fig. 2 presented a realization of block-diagram on Fig. 1 on base of operational voltage amplifiers.

The main disadvantage of the existing heuristic approach to realization is absence to clarity of transition from the block-diagram to the electronic network. Moreover, the usage of the traditional precision elements at realization of block-diagrams inevitably leads to increase of inaccuracy. The expression of amplification coefficient of the diagram on Fig. 2 will be corresponding to expression (1) for the initial block-diagram only when requirements of adjustment have been performing. The usage of requirements of adjustment is necessary even if influence of input resistances and output resistances of amplifiers of the voltage is negligible.

For example, the transfer function of the electronic network on Fig. 2 corresponds to transfer function (1) of block-diagram on Fig. 1 at \( \mu = Z_1/(Z_1+Z_2) \) and \( \beta = 1 \) (the requirements of the adjustment)

\[ S = \frac{K_1(Z_1+Z_2)+K_2(Z_1+Z_2)-K_1 K_2}{(Z_1+Z_2)}. \]  

(2)

The suggested method of the transconductance realization allows to create the electronic networks on base of modern elements, which will be having the transfer function that will be an identical for transfer function of the initial block-diagram.

II. DESCRIPTION OF THE METHOD

In the field of network analysis the transconductance amplifier is defined as an non-ideal voltage controlled current source (VCCS) [2, 3]. A transconductance amplifier (\( G_m \) amplifier) outputs a current \( I_{out} \) proportional to its input voltage \( \Delta U_{in} \)

\[ I_{out} = \Delta U_{in} G_m. \]  

(3)
There are two applications mode of the transconductance amplifier: as a differential amplifier (line 1–2 of Table 1) and as a voltage controlled resistor (line 3 of Table 1).

For the substantiation of the method of the transconductance realization is necessary to consider the representation of the block-diagram of the device in form of signal-flow graph [4, 5], with following isomorphic realization of its elements by the transconductance amplifiers.

The arbitrary block-diagram of the electronic device can be represented by means of the signal-flow graph (column 1–2 of Table 1). Signal-flow graph – is a conditional graphic representation of a set of the equations, which is expressed in cause-and-effect form. The variables of a set of equations (voltage, current) are presented as the node of the graph, and the coefficients are presented as the arcs of the graph. Each arc has a direction and is characterized by the transfer constant. The transfer functions of signal-flow graph can be gotten by means of formula (rule of loops) [4].

In general view the set of equations of the signal-flow graph is of the form of [5]

\[ X = A X + B X_0, \]

where \( X \) – the vector of the unknown variables of dimensionality \( n \), \( X_0 \) – the vector of the known variables of dimensionality \( m \), \( A = [a_{ij}] \) – the matrix of the transfer constants of the arcs \( n \times n \), \( B = [b_j] \) – the matrix of unit transfer constants from source \( n \times m \). In the implicit form the expression (4) is presented in the form of

\[ (I - A)X = B X_0, \]

where \( I \) – unit matrix. Note, that presence of the units on the leading diagonal of matrix in the expression (5) imply presence of the loops with unit transfer constant in the graph. But for signal-flow graphs the presentation of such loops is not necessary.

For example, on Fig. 3,a is presented the classic block-diagram of the amplifier with negative feedback that was described on the Fig. 3,b by the signal-flow graph (variables \( x_1 \) and \( x_2 \) have dimensionality of the voltage). This graph represents the following set of equations in the cause-and-effect form:

\[ x_1 = 1 - \beta x_2, \quad x_2 = K x_1, \]

or in matrix form according to expression (4)

\[ \begin{bmatrix} 1 & \beta \\ -K & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \]

Fig. 3. The block-diagram of the amplifier with negative feedback (a), its signal-flow graph (b) and the transconductance realization (c)

The transfer function of the signal-flow graph on Fig. 3,a will is: \( S = K/(1+K\beta) \). Similar result can be gotten from solution of the set of equations (7) by Kramer’s rule

\[ x_2 = \begin{bmatrix} 1 & 1 \\ -K & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta \\ 1 \end{bmatrix} = \frac{K}{1+K\beta}. \]

It obviously that the representation of the block-diagrams of the electronic networks by the matrix (5) with the units on

<table>
<thead>
<tr>
<th>№</th>
<th>Elements of block-diagram</th>
<th>Elements of signal-flow graph</th>
<th>Realization on the transconductance amplifiers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image" alt="K" /> or <img src="image" alt="K" /></td>
<td>Arc with positive parameter</td>
<td>Noninverting amplifier</td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="β" /></td>
<td>Arc with negative parameter</td>
<td>Inverting amplifier</td>
</tr>
<tr>
<td>3</td>
<td>Summator or node <img src="image" alt="X" /> or <img src="image" alt="X" /></td>
<td>Node <img src="image" alt="X" /></td>
<td>Resistor</td>
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It obviously that the representation of the block-diagrams of the electronic networks by the matrix (5) with the units on
leading diagonal allows to use the conventional matrix approach for analysis. At the transconductance realization of the block-diagram it is important that there is the possibility to present any square matrix of the order \( n \), in the form of the equivalent electronic network based on VCCS [6]. The electronic network have \( n+1 \) nodes (node 0 is grounded node).

Each diagonal element \( ii \) of the matrix is represented on the network by conductivity between nodes \( i \) and 0.

The non-diagonal element \( ij \) of the matrix is corresponds by VCCS, which generator is directed from node \( i \) to node 0, and receiver is directed from node \( j \) to node 0. Parameters of elements of the network are equal to corresponding values of elements of a matrix.

It allow to solve a set of equations in the circuitry-algebraic form on base of its presentation by electronic network [7,8] and avoid the usage of the Kramer’s rule. Under the Generalized Parameter Extraction Method [6–8] the symbol expression of the response for the circuit on Fig. 4,a can be found by the formula

\[
U = \Delta N / \Delta D,
\]

where \( \Delta N, \Delta D \) – the determinants of the circuits, expressed of numerator and denonimn respectively. The Generalized Parameter Extraction Method based on the results [9–13].

\[
\begin{pmatrix}
1 & \beta x_2 & 1 & 1 & 0 \\
0 & 0 & 1 & \beta x_1 & 1 \\
-\beta x_1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

\[
\Delta N =
\begin{pmatrix}
1 & \beta x_2 & 1 & 1 & 0 \\
0 & 0 & 1 & \beta x_1 & 1 \\
-\beta x_1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & \beta x_2 & 1 & 1 & 0 \\
0 & 0 & 1 & \beta x_1 & 1 \\
-\beta x_1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

\[
\Delta D =
\begin{pmatrix}
1 & \beta x_2 & 1 & 1 & 0 \\
0 & 0 & 1 & \beta x_1 & 1 \\
-\beta x_1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

\[
\Delta D =
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0 & 1 & 0 & 1 & 0 \\
\end{pmatrix}
\]

So, the symbol expression of the response for the circuit on Fig. 4,a \( U = K / (1 + K \beta) \) (for \( J = 1 \)).

In general event, each nonzero \( i \)-th element of the vector \( BX_0 \) in the set of equations (5) is expressed on the networks by the current source directed from the node 0 to the node \( i \). The receiver of required voltage is directed from the respective node to the node 0.

Thereby, the representation of a set of equations by the electronic network provides its solution in the analog form without the usage of the Kramer’s rule. Note, that the transconductance amplifiers are used as the simulative element instead of operational amplifiers, which used in traditional analog simulation.

For the transconductance realization of the block-diagram on Fig. 4,a are used both the inverting (see Fig. 4,b) and the noninverting amplifiers (see Fig. 4,c). The expression of the diagonal element of matrix on the circuit requires the inverting amplifiers, in accordance with orientation of VCCS (see Fig. 4,b).

At the representation of the set of equations (5), also as the signal-flow graph, the sign of the elements not belonging to the leading diagonal is changing. Hence, the arcs with the positive transfer constants are expressing by the noninverting amplifier, the arcs with negative transfer constants by the inverting amplifier, and the diagonal elements by the resistor, implemented by connecting the input and output of inverting amplifiers. Thereby, it is possible to present the signal-flow graph directly in the form of transconductance amplifiers based circuits, but with appearance of a certain inaccuracy in consequence of unideality.

Is supposed that the electronic networks based on the transconductance amplifiers will be connected to the current source. For deriving of the potential network, respective to circuit on Fig. 2, is necessary to use the equivalent transformation of the current source to the EMF source. That will brings about the appearance of the new input node.

The representation of the block-diagram in the form of signal-flow graph is an intermediate stage of the transconductance realization and is necessary, first of all, for understanding of the proposed method. For deriving transconductance amplifiers based circuits only the block-diagrams by itself is enough.

The algorithm of the transconductance realization in accordance with Table 1 (first and the third column) will be following: 1) initiating of the main nodes corresponding to the nodes and the summators of the block-diagram; 2) initiating of the new input node in result of transformation of
the current source to the EMF source; 3) adding of the transconductance amplifiers, implemented as conductivity of 1 cm (line 3 of Table 1), between the input node or node of the block-diagram and the new input node; 4) adding of the noninverting amplifiers respective to the elements of the block-diagram with positive transfer constant (line 1 of Table 1); 5) adding of the noninverting amplifiers respective to the elements of the block-diagram with negative transfer constant (line 2 of Table 1); 6) adding of the transconductance amplifiers, implemented as conductivity of 1 cm, between the main nodes (to the exclusion of new input nodes) and grounded node, in accordance with the nodes or the summator of the block-diagram (line 3 of Table 1).

For example, the result of the transconductance realization of the block-diagram on Fig. 3.a is presented on Fig. 3.c, where $G_{m1} = K$, $G_{m2} = \beta$, $G_{m3} = 1$.

The proposed method has been realized in the symbolic circuits analysis program SCAD (http://www.levul.org/sm/symbolic.htm). The program is free distributed.

III. EXAMPLE

The transconductance realization of the block-diagram on Fig. 1 is presented on Fig. 5. We shall obtain the network function of the realized electronic networks based on the transconductance amplifiers, with usage of the Generalized Parameter Extraction Method [6–8] or program SCAD, and compare the result with the transfer constant of the initial block-diagram.

![Fig. 5. The transconductance realization of block-diagram on Fig. 1](image)

For the circuit on Fig. 5 the result will be following:

$$U = G_1 G_{m1} G_{n5} + G_{m2} (G_{m5} G_{n5} - G_{m1} G_{m3}) G_{n5},$$

(12)

where $G_{m1} = K_1$, $G_{m2} = K_2$, $G_{m3} = \mu_1$, $G_{m4} = \beta$, $G_{m5} = 1$.

The derived expression (12) is corresponding with transfer function (1) of the block-diagram of the amplifier on Fig. 1.

IV. CONCLUSIONS

1. The proposed method provides realization of block-diagrams on transconductance amplifiers based circuits, what granting «the second life» to many successful block-diagrams of the past years.

2. Unlike the traditional methods, the offered method allows to derive the transfer function of the electronic network based on the transconductance amplifiers completely identical to the transfer function of the initial block-diagram, without usage of the requirements of the adjustment.

3. The transconductance representation allows to solve systems of the linear algebraic equations of any physical nature, both in digital, and in analogue form.

REFERENCES


